A New Concept Map Model for E-learning Environments

Antonina Dattolo[1⋆] and Flaminia L. Luccio[2]

[1] Dipartimento di Matematica e Informatica, Università di Udine,
Via delle Scienze 206, 33100 Udine, Italy
antonina.dattolo@uniud.it,
[2] Dipartimento di Informatica, Università Ca' Foscari Venezia,
Via Torino 155, 30172 Venezia, Italy
luccio@unive.it

Abstract. Web-based education enables learners and teachers to access a wide quantity of continuously updated educational sources. In order to support the learning process, a system has to provide some fundamental features, such as simple mechanisms for the identification of the collection of "interesting" documents, adequate structures for storing, organizing and visualizing these documents, and appropriate mechanisms for creating personalized adaptive paths and views for learners.
Adaptive Educational Hypermedia seek to apply the personalized possibilities of Adaptive Hypermedia to the domain of education, thereby granting learners a lesson individually tailored to them. A fundamental part of these systems are the concept spaces, i.e., simple and clear visual layouts of concepts and relations among them.
In this paper we propose a new visual layout model in e-learning environments based on the zz-structures, which are graph-centric views capable of representing contextual interconnections among different information. In order to describe the use of these structures, we present their formal analytic description in terms of graph theory, focussing, in particular, on the formal description of two views (H and I views), and on different extensions of these notions to a number $n > 2$ of dimensions. We then apply all these formal descriptions, and some particular properties of zz-structures, to an example in the Web-based education field.

Key words: Adaptive educational hypermedia, concept maps, zz-structures, graph theory, e-learning.

1 Introduction

In the last decades, a great effort has been devoted to the diffusion of knowledge through the Web. The field of education has evolved towards this new direction, developing the so called Web-based education techniques, which enable teachers and learners to interact and exchange continuously updated educational sources.
Web-based education techniques are used in Adaptive Educational Hypermedia (AEH);

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these last systems apply standard techniques of Adaptive Hypermedia systems [1] to the domain of education: main task of these systems are the creation and diffusion of personalized learning material in order to grant lessons individually tailored to the learners [3]. A fundamental part of an AEH are the concept spaces [7]: they provide an ontology of the subject matter including the concepts and their relationships to one another.
Concept spaces are traditionally visualized using a concept map diagram, a downward-branching, hierarchical tree structure, which, in mathematical terms, can be represented as a directed acyclic graph, a generalization of a tree structure, where certain sub-trees can be shared by different parts of the tree.
Concept maps have got the double advantage of visually representing an information map and linking it to useful material contained in a database. Learners have a referring map to which they can come back to review previous steps, and, mostly, learn how to organize information so "it makes sense" for them. Thus, the main purpose of concept mapping is the representation of visual layouts that clarify concepts and not the production of general maps that only represent at a very high level the relations among them.

Related Works. In the literature different solutions based on traditional concept maps have been proposed [8], and suite very well small collections of information. However, they are inadequate to capture and visualize a very large amount of information. Some work has been done towards this direction with the proposal of more innovative tree visualization techniques which, on the other hand, are not well suited to represent concept maps: for example Shneiderman's Treemaps [22] and Kleiberg's Botanical trees [9] cannot easily differentiate between relationship types; other models (e. g. [5], based on hyperbolic geometry, or [23], based on S-nodes are not able to dynamically switch from a view to another one. It is often not possible to view the entire concept space on-screen without zooming out so far that the concept and relationship labels are no longer readable. Similarly, the large number of relationships improve the difficulty of understanding the structure of the concept space.
In particular, in the e-learning field, there are many reasons to define opportune structure models for storing and visualizing concept maps:

- They allow the system to be adaptive: current approaches and tools (see WebCT, Moodle, etc.) neither support a comprehensive analysis of users' needs, demands and opportunities, nor they support a semantic analysis of texts, thus they are not adaptive.
- They provide interoperability between different adaptive systems: this feature becomes not only desirable but also necessary, as it enables the re-use of previously created material without the cost of recreating it from scratch [6].
- They simplify the authoring process, in which the user/learner may assume the role of an author (see, e.g., Wikis and Wiki farms).

For all these reasons, in our opinion, it is important to propose new models which better suite the requested requirements. We will thus focus our attention on an innovative structure, proposed in [21], the zz-structure, that constitutes the main part of a ZigZag system [20].
Previous work in this direction has shown how flexible this structure is, and how it can

be specialized in different fields. It has been used for the modeling, e.g., of an information manager for mobile phones (zz-phones) [18], of the London underground train lines and stations [20], of bioinformatics workspaces [19], of data grid systems [10], of virtual museum tours [12], of an authoring system for electronic music (Archimedes) [13]. This structure has also been used in Web-based courses [4] in order to establish attribute-based connections among Web documents retrieved by authors using search engines.
Starting from [21] and the other previously mentioned works many informal descriptions of this structure have been provided, and some preliminary formal description has been proposed in [16]. However, in our opinion, a formal and complete description of the structure may be very useful in simplifying the comprehension of the model. Nelson itself writes: "The ZigZag system is very hard to explain, especially since it resembles nothing else in the computer field that we know of, except perhaps a spreadsheet cut into strips and glued into loops ".

Contributions of this Work. The general goal of this work is to propose a formal structure for representing and visualizing a concept space. This model is based both on zz-structures and on graph theory.

Our application field is Web-based education, in which *learners* and *authors* (teachers) have to access a wide quantity of continuously updated educational sources. The learning process of learners, and the course creation/modification/organization process of authors, can be greatly simplified by providing them tools to:

1. identify the collection of "interesting" documents, for example applying semantic filtering algorithms [2], or proximity metrics on the search engine results [4];
2. store the found collection of documents in adequate structures, that are able to organize and visualize concept spaces;
3. create personalized adaptive paths and views for learners.

These three topics are the guidelines of our current research. In this paper, we focus our attention only on point 2. We assume that an author has a collection of available documents on a given topic that have to be organized in concept maps, suitable for different learners. E.g., some users could be could be doing research on a specific research area, others could be preparing a degree thesis, and so on. Thus, authors need adequate tools to organize documents in a concept space, and to create semantic interconnections and personalized maps.

We will show how identifying and defining in an analytic way the graph theoretical structure of zz-structures can both provide interesting insights to educational hypermedia designers (facilitating a deeper understanding of which model might best support the representation and interaction aims of their systems), and to learners (offering them support for Web orientation and navigation).

Summarizing, the novel contributions of this work are:

– a formal analytic graph-based description of zz-structures. Particular attention has been devoted to the formalization of two views (H and I views), present into all ZigZag implementations;

- different extensions of the concept of H and I views from a number 2 towards a number $n > 2$ of dimensions;
- a new concept map model for e-learning environments, based on our model.

The paper is organized as follows: in Section 2, we introduce the reader to zz-structures and we present some basic graph theory definitions; in Section 3, we propose our formal definition of zz-structures, and we use these structures as a reference model for representing concept maps. Finally, in Section 4 we first introduce the definition of the standard H and I views, and then we extend this definition to the non-standard n-dimensions views (with $n > 2$). Conclusion and future works conclude the paper.

2 Zz-structures and Graph Theory

This section is introduced for consistency. If the reader has a background on the ZigZag model and on basic graph theory, can skip it.

2.1 An Introduction to Zz-structures

Zz-structures [21] introduce a new, graph-centric system of conventions for data and computing. A zz-structure can be thought of as a space filled with cells. Each cell may have a content (such as integers, text, images, audio, etc.), and it is called *atomic* if it contains only one unit of data of one type [19], or it is called *referential* if it represents a package of different cells. There are also special cells, called *positional*, that do not have content and thus have a positional or topographical function.

Cells are connected together with links of the same color into linear sequences called *dimensions*. A single series of cells connected in the same dimension is called *rank*, i.e., a rank is in a particular dimension. Moreover, a dimension may contain many different ranks. The starting and an ending cell of a rank are called, *headcell* and *tailcell*, respectively, and the direction from the starting (ending) to the ending (starting) cell is called *posward* (respectively, *negward*). For any dimension, a cell can only have one connection in the posward direction, and one in the negward direction. This ensures that all paths are non-branching, and thus embodies the simplest possible mechanism for traversing links. Dimensions are used to project different structures: ordinary lists are viewed in one dimension; spreadsheets and hierarchical directories in many dimensions.

The interesting part is how to view these structures, i.e., there are many different ways to arrange them, choosing different dimensions and different structures in a dimension. A *raster* is a way of selecting the cells from a structure; a *view* is a way of placing the cells on a screen. *Generic views* are designed to be used in a big variety of cases and usually show only few dimensions or few steps in each dimension. Among them the most common are the *two-dimensions rectangular views*: the cells are placed, using different rasters, on a Cartesian plane where the dimensions increase going down and to the right. Obviously some cells will not fit in these two dimensions and will have to be omitted. The simplest raster is the row and column raster, i.e., two rasters which are the same but rotated of 90 degrees from each other. A cell is chosen and placed at the center of the plane (cursor centric view). The chosen cell, called focus, may be changed by moving the cursor horizontally and vertically. In a row view I, a rank is chosen and placed vertically. Then the ranks related to the cells in the vertical rank are placed horizontally. Vice versa, in the column view H, a rank is chosen and placed horizontally and the related ranks are placed vertically. All the cells are denoted by different numbers. Note that in a view the same cell may appear in different positions as it may represent the intersection of different dimensions.

2.2 Basic Graph Theory Definitions

In the following we introduce some standard graph theory notation (see also [15]).
A *graph* G is a pair $G = (V,E)$, where V is a finite non-empty set of elements called *vertices* and E is a finite set of distinct unordered pairs $\{u,v\}$ of distinct elements of V called *edges*.
A *multigraph* is a triple $MG = (V,E,f)$ where V is a finite non-empty set of vertices, E is the set of edges, and $f : E \rightarrow \{\{u,v\} \mid u,v \in V, u \neq v\}$ is a surjective function.
An *edge-colored multigraph* is a triple $ECMG = (MG,C,c)$ where: $MG = (V,E,f)$ is a multigraph, C is a set of colors, $c : E \rightarrow C$ is an assignment of colors to edges of the multigraph.
In a multigraph $MG = (V,E,f)$, edges $e_1,e_2 \in E$ are called *multiple* or *parallel* iff $f(e_1) = f(e_2)$. Thus, a graph as a particular multigraph without parallel edges.
Given an edge $e = \{u,v\} \in E$, we say that e is *incident* to u and v; moreover u and v are *neighboring* vertices. Given a vertex $x \in V$, we denote with $deg(x)$ its degree, i.e., the number of edges incident to x, and with d_{max} the maximum degree of the graph, i.e., $d_{max} = \max_{z \in V}\{deg(z)\}$. In an edge-colored (multi)graph ECMG, where $c_k \in C$, we define $deg_k(x)$ the number of edges of color c_k incident to vertex x. A vertex of degree 0 is called *isolated*, a vertex of degree 1 is called *pendant*.
A *path* $P = \{v_1,v_2,\ldots,v_s\}$ is a sequence of neighboring vertices of G, i.e., $\{v_i,v_{i+1}\} \in E$, $1 \leq i \leq s-1$. A graph $G = (V,E)$ is *connected* if: $\forall x,y \in V$, $\exists$ a path $P = \{x = v_1,v_2,\ldots,v_s = y\}$, with $\{v_k,v_{k+1}\} \in E$, $1 \leq k \leq s-1$. Two vertices x and y in a connected graph are at *distance d* if the *shortest path* connecting them is composed of exactly d edges.
Finally, a $m \times n$ *mesh* is a graph $M_{m,n} = (V,E)$ with $v_{i,j} \in V$, $0 \leq i \leq m-1$, $0 \leq j \leq n-1$, and E contains exactly the edges $(v_{i,j},v_{i,j+1})$, $j \neq n-1$, and $(v_{i,j},v_{i+1,j})$, $i \neq m-1$.

3 The Formal Model

In this section, we formalize the model presented in [21] in terms of graph theory. In the rest of this paper we describe formal definitions through a simple example in the e-learning field: an author has a collection of different material (e.g., books, articles, etc.) that first wants to link through different semantic paths and then wants to merge into a unique concept space. Books that have been published by the same publisher, or books on a related topic, or books that share one author, are examples of semantic paths, which automatically generate concept maps.

3.1 Zz-structures

A zz-structure can be viewed as a multigraph where edges are colored, with the restriction that every vertex has at most two incident edges of the same color. Differently from [16], but as mentioned in [17, 10], we consider undirected graphs, i.e., edges may be traversed in both directions. A *zz-structure* is formally defined as follows.

Definition 1. *(***Zz-structure***). A* zz-structure *is an edge-colored multigraph* $S = (MG, C, c)$*, where* $MG = (V, E, f)$*, and* $\forall x \in V$, $\forall k = 1, 2, ..., |C|$, $deg_k(x) = 0, 1, 2$. *Each vertex of a zz-structure is called* zz-cell *and each edge* zz-link. *The set of isolated vertices is* $V_0 = \{x \in V \ : deg(x) = 0\}$.

An example of a zz-structure is given in Fig. 1. The structure is a graph, where vertices $v_1, \ldots, v_{14}$ represent different books, and edges of the same kind represent the same semantic connection. In particular, in this example, thick edges connect a sequence of

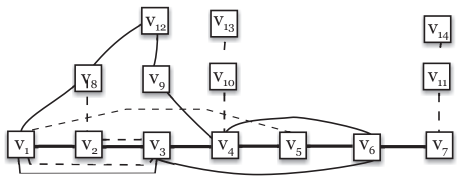

Fig. 1: A zz-structure where thick, normal and dotted lines represent three different colors.

books published by the same publisher (e.g., Elsevier), dotted edges group books that have at least an author in common, finally, normal lines link books on the same topic (e.g., hypermedia, algorithms, etc.).

3.2 Dimensions

An alternative way of viewing a zz-structure is a union of subgraphs, each of which contains edges of a unique color.

Proposition 1. *Consider a set of colors* $C = \{c_1, c_2, ..., c_{|C|}\}$ *and a family of indirect edge-colored graphs* $\{D^1, D^2, ..., D^{|C|}\}$*, where* $D^k = (V, E^k, f, \{c_k\}, c)$*, with* $k = 1, ..., |C|$*, is a graph such that: 1)* $E^k \neq \emptyset$*; 2)* $\forall x \in V$, $deg_k(x) = 0, 1, 2$.

Then, $S = \bigcup_{k=1}^{|C|} D^k$ is a zz-structure.

Definition 2. *(***Dimension***). Given a zz-structure* $S = \bigcup_{k=1}^{|C|} D^k$*, then each graph* D^k*,* $k = 1, \ldots, |C|$*, is a distinct* dimension *of* S.

From Fig. 1 we can extrapolate three dimensions, one for each different color (i.e., one for each different semantic connection). As shown in Fig. 2, we associate thick lines to dimension D^{book}, dotted lines to dimension D^{author}, and normal lines to dimension D^{topic}. Each dimension can be composed of isolated vertices (e.g., vertices v_6, v_9, v_{12} in dimension D^{author}), of distinct paths (e.g., the three paths $\{v_8, v_2, v_3, v_1, v_5\}$, $\{v_4, v_{10}, v_{13}\}$ and $\{v_7, v_{11}, v_{14}\}$ in dimension D^{author}), and of distinct cycles (e.g., the unique cycle $\{v_1, v_3, v_6, v_4, v_9, v_{12}, v_8, v_1\}$ in dimension D^{topic}).

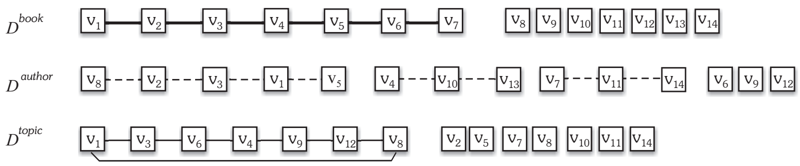

Fig. 2: The three dimensions.

3.3 Ranks

Definition 3. *(***Rank***). Consider a dimension $D^k = (V, E^k, f, \{c_k\}, c)$, $k = 1, \ldots, |C|$ of a zz-structure $S = \cup_{k=1}^{|C|} D^k$. Then, each of the l_k connected components of D^k is called a* rank.

Thus, each rank $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$, $i = 1, \ldots, l_k$, is an indirect, connected, edge-colored graph such that: 1) $V_i^k \subseteq V$; 2) $E_i^k \subseteq E^k$; 3) $\forall x \in V_i^k$, $1 \leq deg_k(x) \leq 2$. A *ringrank* is a rank R_i^k, where $\forall x \in V_i^k,\ deg_k(x) = 2$.

Note that the number l_k of ranks differs in each dimension D^k, e.g. in Fig. 2, dimension D^{author} has three ranks ($\{v_8, v_2, v_3, v_1, v_5\}$, $\{v_4, v_{10}, v_{13}\}$ and $\{v_7, v_{11}, v_{14}\}$), and dimension D^{book} has a unique rank ($\{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$). A ringrank is, e.g., the cycle $\{v_1, v_3, v_6, v_4, v_9, v_{12}, v_8, v_1\}$ of dimension D^{topic}.

Definition 4. *(Parallel ranks). Given a zz-structure $S = \cup_{k=1}^{|C|} D^k$, m ranks $R_j^k = (V_j^k, E_j^k, f, \{c_k\}, c)$, $(j = 1, 2, \ldots, m,\ 2 \leq m \leq l_k)$ are* parallel ranks *on the same dimension D^k, $k \in \{1, \ldots, |C|\}$ iff $V_j^k \subseteq V$, $E_j^k \subseteq E^k$, $\forall j = 1, 2, \ldots, m$, and $\cap_{j=1}^{m} V_j^k = \emptyset$.*

In Fig. 2 the three ranks of dimension D^{author} are parallel.

3.4 Cells and their Orientation

A vertex has local orientation on a rank if each of its (1 or 2) incident edges has assigned a distinct label (1 or -1). More formally (see also [14]):

Definition 5. *(Local orientation). Consider a rank $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$ of a zz-structure $S = \cup_{k=1}^{|C|} D^k$. Then, $\exists$ a function $g_x^i : E_i^k \rightarrow \{-1, 1\}$, such that, $\forall x \in V_i^k$, if $\exists y, z \in V_i^k : \{x, y\}, \{x, z\} \in E_i^k$, then $g_x^i(\{x, y\}) \neq g_x^i(\{x, z\})$. Thus, we say that each vertex $x \in V_i^k$ has a* local orientation *in R_i^k.*

Definition 6. *(Posward and negward directions). Given an edge $\{a, b\} \in E_i^k$, we say that $\{a, b\}$ is in a* posward *direction from a in R_i^k, and that b is its* posward cell *iff $g_a^i(\{a, b\}) = 1$, else $\{a, b\}$ is in a* negward *direction and a is its* negward cell*. Moreover, a path in rank R_i^k follows a posward (negward) direction if it is composed of a sequence of edges of value 1 (respectively, -1).*

For simplicity, given a rank R_i^k, the notation $\ldots x^{-2} x^{-1} x x^{+1} x^{+2} \ldots$, where x^{-1} represents the negward cell of x and x^{+1} the posward cell, describes the path composed by a sequence of x's negward cells, by the vertex x and by a sequence of x's posward cells, Thus, x^{-i} (x^{+i}) is a cell at distance i in the negward (posward) direction, and $x^0 = x$.

Definition 7. *(Headcell and tailcell). Given a rank $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$, a cell x is the* headcell *of R_i^k iff $\exists$ its posward cell x^{+1} and $\nexists$ its negward cell x^{-1}. Analogously, a cell x is the* tailcell *of R_i^k iff $\exists$ its negward cell x^{-1} and $\nexists$ its posward cell x^{+1}.*

4 Views

We now formalize the standard notion of H and I views in two dimensions, and we then propose a new definition of H and I-views in n dimensions. We also show some interesting applications of these new higher dimensional views.

In the following, $x \in R^a_{(x)}$ denotes the rank $R^a_{(x)}$ related to vertex x of color c_a.

Two Dimensions Views. Standard two dimensional views may be considered H and I views.

Definition 8. *(H-view). Given a zz-structure $S = \cup_{k=1}^{|C|} D^k$, where $D^k = \cup_{i=1}^{l_k}(R_i^k \cup V_0^k)$, and where $R_i^k = (V_i^k, E_i^k, f, \{c_k\},\ c)$, the H-view of size $l = 2m+1$ and of* focus *$x \in V = \cup_{i=0}^{l_k} V_i^k$, on main vertical dimension D^a and secondary horizontal dimension D^b $(a, b \in \{1, ..., l_k\})$, is defined as a tree whose embedding in the plane is a partially connected colored $l \times l$ mesh in which:*

- *the central node, in position $((m+1),(m+1))$, is the focus x;*
- *the horizontal central path (the $m+1$-th row) from left to right, focused in vertex $x \in R^b_{(x)}$ is: $x^{-g} \dots x^{-1} x x^{+1} \dots\ x^{+p}$ where $x^s \in R^b_{(x)}$, for $s = -g, \dots, +p$ $(g, p \le m)$.*
- *for each cell x^s, $s = -g, \dots, +p$, the related vertical path, from top to bottom, is: $(x^s)^{-g_s} \dots (x^s)^{-1} x^s (x^s)^{+1} \dots (x^s)^{+p_s}$, where $(x^s)^t \in R^a_{(x^s)}$, for $t = -g_s, \dots, +p_s$ $(g_s, p_s \le m)$.*

Intuitively, the H-view extracts ranks along the two chosen dimensions. Note that, the name H-view comes from the fact that the columns remind the vertical bars in a capital letter H. Observe also that the cell x^{-g} (in the $m+1$-th row) is the *headcell* of $R^b_{(x)}$ if $g < m$ and the cell x^{+p} (in the same row) is the *tailcell* of $R^b_{(x)}$ if $p < m$. Analogously, the cell x^{-g_s} is the headcell of $R^a_{(x^s)}$ if $g_s < m$ and the cell x^{+p_s} is the tailcell of $R^a_{(x^s)}$ if $p_s < m$. Intuitively, the view is composed of $l \times l$ cells unless some of the displayed ranks have their headcell or tailcell very close (less than m steps) to the chosen focus.

As an example consider Fig. 3 left that refers to the zz-structure of Fig. 1. The main vertical dimension is D^{author} and the secondary horizontal dimension is D^{book}. The view has size $l = 2m+1 = 5$, the focus is v_3, the horizontal central path is $v_3^{-2} v_3^{-1} v_3 v_3^{+1} v_3^{+2} = \{v_1, v_2, v_3, v_4, v_5\}$ $(g, p = 2)$. The vertical path related to $v_3^{-1} = v_2$ is $(v_3^{-1})^{-1}(v_3^{-1})(v_3^{-1})^{+1}(v_3^{-1})^{+2} = \{v_8, v_2, v_3, v_1\}$ $(g_s = 1$ and $p_s = 2)$, that is $(v_3^{-1})^{-1} = v_8$ is the headcell of the rank as $g_s = 1 < m = 2$.

Analogously to the H-view we can define the I-view.

Definition 9. *(I-view). Given a zz-structure $S = \cup_{k=1}^{|C|} D^k$, where $D^k = \cup_{i=1}^{l_k}(R_i^k \cup V_0^k)$, and where $R_i^k = (V_i^k,\ E_i^k, f, \{c_k\},\ c)$, the I-view of size $l = 2m+1$ and of* focus *$x \in$*

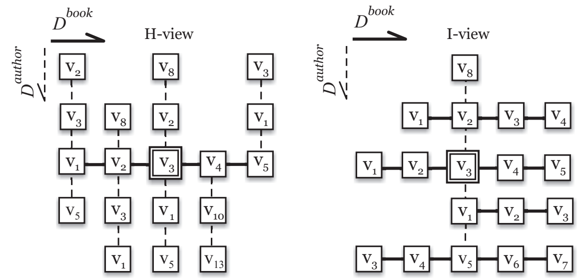

Fig. 3: H-view and I-view, related to Fig. 1.

$V = \cup_{i=0}^{l_k} V_i^k$ on main horizontal dimension D^a and secondary vertical dimension D^b $(a, b \in \{1, ..., l_k\})$, is defined as a partially connected colored $l \times l$ mesh in which:

- *the central node, in position $((m+1),(m+1))$ is the focus x;*
- *the vertical central path (the $m+1$-th column) from top to bottom,focused in vertex $x \in R^b_{(x)}$ is: $x^{-u} \dots x^{-1} x x^{+1} \dots x^{+r}$ where $x^s \in R^b_{(x)}$, for $s = -u, \dots, +r$ $(u, r \le m)$.*
- *for each cell x^s, $s = -u, \dots, +r$, the related horizontal path, from left to right, is: $(x^s)^{-u_s} \dots (x^s)^{-1} x^s (x^s)^{+1} \dots (x^s)^{+r_s}$, where $(x^s)^t \in R^a_{(x^s)}$, for $t = -u_s, \dots, +r_s$ $(u_s, r_s \le m)$.*

Note that, the name I-view comes from the fact that the rows remind the horizontal serif in a capital letter I. Observe also that the cell x^{-u} (in the $m+1$-th column) is the *headcell* of $R^b_{(x)}$ if $u < m$ and the x^{+r} (in the same column) is the *tailcell* of $R^b_{(x)}$ if $r < m$. Analogously, the cell x^{-u_s} is the headcell of $R^a_{(x^s)}$ if $u_s < m$ and the x^{+r_s} is the tailcell of $R^a_{(x^s)}$ if $r_s < m$.

As example consider Fig. 3 right. The main horizontal dimension is D^{book} and the secondary vertical dimension is D^{author}. The view has size $l = 2m+1 = 5$, the focus is v_3, the vertical central path is $v_3^{-2} v_3^{-1} v_3 v_3^{+1} v_3^{+2} = \{v_8, v_2, v_3, v_1, v_5\}$ $(u, r = 2)$. The horizontal path related to $v_3^{-1} = v_2$ is $(v_3^{-1})^{-1} \dots (v_3^{-1})^{+2} = \{v_1, v_2, v_3, v_4\}$ (i.e., $r = 2$). Vice versa the horizontal path related to $v_3^{+1} = v_1$ is $\{v_1, v_2, v_3\}$ and v_1 is the headcell. Finally, the horizontal path related to $v_3^{+2} = v_5$ is $\{v_3, v_4, v_5, v_6, v_7\}$.

n-Dimensions Views. We can now extend the known definition of H and I views to a number $n > 2$ of dimensions. Intuitively, we will build $n-1$ different H-views (respectively, I-views), centered in the same focus, with a fixed main dimension and a secondary dimension chosen among the other $n-1$ dimensions. Formally:

Definition 10. *(n-dimensions H-view). Given a zz-structure $S = \cup_{k=1}^{|C|} D^k$, where $D^k = \cup_{i=1}^{l_k} (R_i^k \cup V_0^k)$, and where $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$, the* n-dimensions H-view *of size $l = 2m+1$ and of* focus s $x \in V = \cup_{i=0}^{l_k} V_i^k$*, on dimensions $D^1, D^2, \dots, D^n$ is composed of $n-1$ rectangular H-views, of main dimension D^1 and secondary dimensions D^i, $i = 2, \dots, n$, all centered in the same focus x.*

Analogously, we have the following:

Definition 11. *(n-dimensions I-view). Given a zz-structure* $S = \cup_{k=1}^{|C|} D^k$*, where* $D^k = \cup_{i=1}^{l_k}(R_i^k \cup V_0^k)$*, and where* $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$*, the n-*dimensions *I*-view *of size* $l = 2m+1$ *and of* focus $x \in V = \cup_{i=0}^{l_k} V_i^k$*, on dimensions* $D^1, D^2, \ldots, D^n$ *is composed of* $n-1$ *rectangular I-views of main dimension* D^1*, and secondary dimensions* D^i*,* $i = 2, \ldots, n$*, all centered in the same focus x.*

In Fig. 3, we can distinguish only two dimensions (D^{book} and D^{author}).

To display a 3-dimensions H-view we can add a new dimension (let it be D^{topic}). This new H-view has main dimension D^{topic}, and secondary dimensions D^{book} and D^{author}. To construct this view we start from Fig. 1 using v_3 as focus, and we consider the two central paths (Fig. 4 left), related to the two secondary dimensions D^{book} and D^{author}.

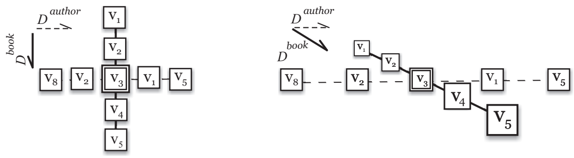

Fig. 4: Two secondary dimensions cross the focus v_3.

The same visualization is shown in Fig. 4 right under a different perspective.
Finally, in Fig. 5 we obtain the 3-dimensions H-view where the vertical paths (on the main dimension D^{topic}) are added.

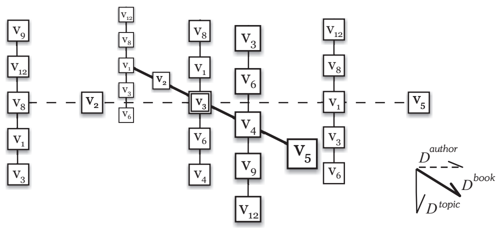

Fig. 5: An example of a 3-dimensions H-view.

We can now extend this example to the n-dimensions case. In Fig. 6, we show a 5-dimensions view, considering four secondary dimensions. In our example, we have added other two dimensions ($D^{publisher\ location}$ and $D^{publication\ year}$), representing the location of the publisher and the year of publication of the article. This new view has focus v_3, size $l = 2m+1 = 5$ and main dimension $D^{publication\ year}$.

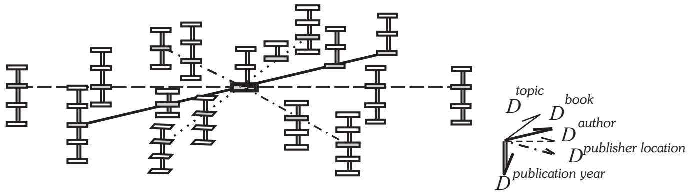

Fig. 6: A 5-dimensions H-view.

3-Dimensions Extended Views. In the 3-dimensions case, we can extend the previous definition of a 3-dimensions H (or I) view. Intuitively, we build a standard 2-dimensions H (or I) view and, starting from each of the related cells as focus, we display also the ranks in the third dimension. Formally:

Definition 12. *(3-dimensions extended H-view) Consider a zz-structure* $S = \cup_{k=1}^{|C|} D^k$*, where* $D^k = \cup_{i=1}^{l_k}(R_i^k \cup V_0^k)$*, and where* $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$*. The* 3-dimensions extended H-view *of size* $l = 2m+1$ *and of* focus $x \in V = \cup_{i=0}^{l_k} V_i^k$*, on dimensions* D^1, D^2, D^3*, is composed as follows:*

– *the central path (the* $m+1$*-th row) from left to right, focused in vertex* $x \in R_{(x)}^3$*:* $x^{-g} \ldots x \ldots x^{+p}$*, where* $x^s \in R_{(x)}^3$*, for* $s = -g, \ldots, +p,\ g, p \leq m$ *and* $g+p+1 = l'$*;*
– l' *rectangular H-views of same size* l *and of focuses respectively* $x^{-g}, \ldots, x, \ldots, x^{+p}$*, on main dimension* D^1 *and secondary dimension* D^2*.*

Analogously we can define a 3-dimensions extended I-view.

Definition 13. *(3-dimensions extended I-view). Consider a zz-structure* $S = \cup_{k=1}^{|C|} D^k$*, where* $D^k = \cup_{i=1}^{l_k}(R_i^k \cup V_0^k)$*, and where* $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$*. The* 3-dimensions extended I-view *of size* $l = 2m+1$ *and of* focus $x \in V = \cup_{i=0}^{l_k} V_i^k$*, on dimensions* D^1, D^2, D^3*, is composed as follows:*

– *the central path (the* $m+1$*-th column) from top to bottom, focused in vertex* $x \in R_{(x)}^3$*:* $x^{-u} \ldots x \ldots x^{+r}$*, where* $x^s \in R_{(x)}^3$*, for* $s = -u, \ldots, +r,\ u, r \leq m$ *and* $u+r+1 = l''$*;*
– l'' *rectangular I-views of same size* l *and of focuses respectively* $x^{-u}, \ldots, x, \ldots, x^{+r}$*, on main dimension* D^1 *and secondary dimension* D^2*.*

As example, we start from Fig. 4 and we consider the related 2-dimensions H-view of size 5 and of focus v_3, on main dimension D^{book} and secondary dimension D^{author}. We obtain the H-view shown in Fig. 7.
Now, we change perspective and, for each cell of this view, we visualize the related ranks in dimension D^{topic} (see Fig. 8).

Star Views. A star view visualizes information related to a focus vertex and a set of n chosen dimensions. We propose a formal definition for two typologies of star views: the *star view* and the *m-extended star view*.

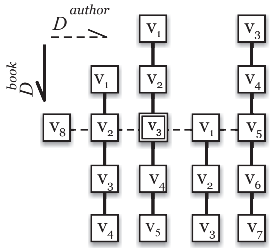

Fig. 7: Standard 2-dimensions *H*-view.

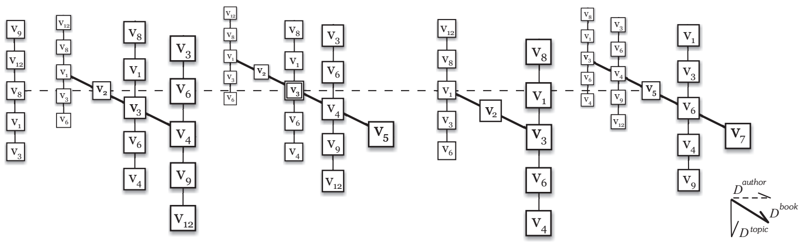

Fig. 8: A 3-dimensions *extended H*-view.

Definition 14. *Given a zz-structure* $S = \bigcup_{k=1}^{|C|} D^k$*, where* $D^k = \bigcup_{i=1}^{l_k} R_i^k \cup V_0^k$*, and where* $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$*, the* star view *of* focus $x \in V = \bigcup_{i=0}^{l_k} V_i^k$ *and dimensions* $D^1, D^2, \ldots, D^n$ *is a star graph* $_{n+1}$*-star on central vertex x and neighborhood* $N(x) = \{y \in V : y = x^{+1}, x^{+1} \in R^i_{(x)}, i \in \{1, \ldots, n\}\}$.

In order to extend the number of documents directly accessible from a view, we introduce the definition of *m-extended star view*; it is based on a star view, but, for each vertex y in the neighborhood $N(x)$, adds the set of the p ($p \leq m$) posward cells related to the given dimensions.

Definition 15. *Given a zz-structure* $S = \bigcup_{k=1}^{|C|} D^k$*, where* $D^k = \bigcup_{i=1}^{l_k} R_i^k \cup V_0^k$*, and where* $R_i^k = (V_i^k, E_i^k, f, \{c_k\}, c)$*, the* m-extended star view *is a* star view *of focus* $x \in V = \bigcup_{i=0}^{l_k} V_i^k$*, dimensions* $D^1, D^2, \ldots, D^n$*, and extension constituted,* $\forall y \in N(x)$ *and* $\forall i \in \{1, \ldots, n\}$*, by the paths* $(y^{+1}, \ldots, y^{+p}) \subseteq R^i_{(x)}$ *(*$p \leq m$*).*

A schematic example of 5-extended star view is shown in Fig. 9. In this case, the central node v_3 represents a person, and the view shows the connections along seven dimensions (D^{topic}, D^{author}, D^{book}, $D^{publication\ year}$, $D^{publisher\ location}$, …).

5 Conclusion

In this paper we have presented a formal model for the representation and the use of concepts maps in the area of Web-based education. This work is part of a larger project; current advances include:

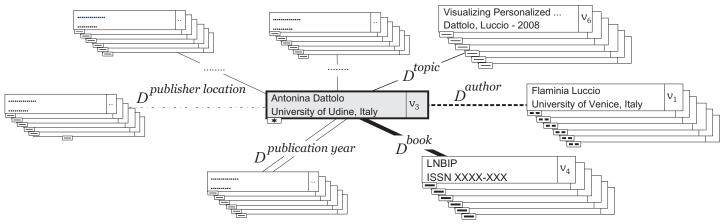

Fig. 9: A 5-extended star view.

– automatic semantic filtering methodologies;
– an extension of this model towards an open, distributed and concurrent agent based architecture;
– adaptive navigation and presentation for learners;
– authoring facilities for web-based courses.

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