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A FUZZY APPROACH TO STUDENT MODELING

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Abstract—After some basic concepts of fuzzy theory are briefly recalled, the problem of evaluating the student's knowledge state is investigated and a suitable fuzzy measure is introduced. The properties of the measure are discussed and a heuristic is illustrated which uses the measure to deal with the student's current learning level as regards a specific topic. Finally, it is shown how this assessment can be used by the teaching module to improve its tutoring strategies.

1. INTRODUCTION

Originating during the 1970s, Intelligent Tutoring Systems (ITSs) aim to overcome the limits of traditional computer assisted instruction systems. This result is achieved by imposing artificial intelligence (AI) techniques on classical teaching methods. The objective of ITSs is to bring more interactivity and flexibility to tutoring domains so that the system can communicate knowledge to the student at the appropriate level. This goal is very exacting and is still a matter of investigation. In the following we present some basic issues to which textbooks such as McFarland and Parker[1] or Polson and Richardson[2] are an introduction.

Although there is no general agreement about the basic structure of an ITS, most researchers distinguish four modules:

Expert module, that has knowledge about the topic to be taught and generates instructional content.
Student model module, that is used to assess the student's knowledge states and to make hypotheses about their conceptions and reasoning strategies.
Tutorial module, that explicates adequate tutoring strategies.
Administrative module, that regulates all activities within the ITS and supports the interface with the student.

It is fairly simple to justify the presence of these modules. In fact, we refer to an educational situation involving a teaching system and a student; the object of tutoring is knowledge in some domain and this has to be presented to the student in the most suitable way.

The emphasis in an ITS is normally placed on the Student Module[3,4]. Different types of representation have been envisioned and they can essentially be partitioned into four classes[5]:

(1) *Performance measure*—this does not indicate what knowledge has been acquired, only how much knowledge.
(2) *Overlay models*—this assumes that the student's knowledge is a subset of the expert's.
(3) *Buggy models*—student knowledge is represented as a set of bugs/misconceptions.
(4) *Simulations*—from a protocol analysis of the student's behavior, an executable model is built. This represents the student's knowledge and can be used to solve the problems presented.

It is worth emphasizing that these types of representation heavily rely on traditional logic; however we note that the learning level is a concept inherently vague and imprecise and as a consequence, student modeling may greatly benefit from the application of fuzzy theory whose specific purpose is to manage such types of concepts and situations.

2. THE THEORY OF FUZZY SETS

The theory of fuzziness aims at managing with concepts and situations that cannot be described in precise terms since they are inherently vague and non-specific. The term "fuzzy" first appeared in a paper by Zadeh[6] where he speaks about "mathematics of *fuzzy* or cloudy quantities which are not describable in terms of probability distributions". Examples drawn from daily life are terms such as tall, beautiful, intelligent which cannot be described in precise terms, or in probabilistic terms. Fuzziness is best expressed as follows[7]:

> "Essentially, fuzziness is a type of imprecision that stems from a grouping of elements into classes that do not have sharply defined boundaries, such classes—called **fuzzy sets**—arise, for example, whenever we describe **ambiguity, vagueness,** and **ambivalence** in mathematical models of empirical phenomena."

In 1965 the concept of fuzzy sets was formalized by Zadeh[8] and in the last two decades the theory of fuzzy sets has been developing steadily. Persons working in various areas consider fuzzy sets as a concern and a tool: disciplines such as logic, AI, decision theory, genetics have already been affected by the theory of fuzzy sets and successful applications have been developed.

Before illustrating how fuzziness can be useful to evaluate the student's cognitive state, we define what a fuzzy set is. Informally a fuzzy set is a class of objects which does not have precisely defined criteria of membership so that membership is no longer an all-or-nothing notion. Transition between membership and nonmembership is gradual rather than abrupt.

A fuzzy set can be formally defined as follows. Let X be a set of objects, whose generic elements are noted by x. Membership in a classical subset A of X is often dealt with a characteristic function $\boldsymbol{\chi}_{\mathbf{A}}(\mathbf{x})$ such that $\chi_A(x) = 1$ iff $x \in A$, 0 otherwise. The set $\{0, 1\}$ is called a valuation set. If the valuation set is allowed to be the real interval [0, 1], A is called a fuzzy set. The function $\chi_A(x)$ is the grade of membership of x in A. Thus, A is a subset of X which has no sharp boundary and the closer the value of $\chi_A(x)$ is to 1, the more x belongs to A.

As a consequence of this definition a fuzzy set A is completely characterized by the set of pairs

$$A = \{(x, \chi_A(x))\}, x \in X\}.$$

It is apparent that also educational systems can benefit from the theory of fuzziness: for example, a student might be "lazy" or "willing" and these are classical fuzzy concepts. Then, as student modeling aims at evaluating the student's performance starting from available data a fuzzy approach can cope with a situation whose elements are inherently imprecise and uncertain.

3. THE FUZZY MEASURE

In order to introduce a suitable fuzzy measure which can be useful for student modeling, we have to consider the set $A = \{x, \chi_A(x)\}$, where x denotes the generic concept and the characteristic function expresses to what extent the concept is mastered by the student; thus essentially each concept gets associated with a fuzzy weight.

The initial values of the weights are assigned by default or can stem from preliminary entry tests. Subsequently weights are dynamically managed and suitably modified according to the student's behavior.

The crucial point is to make hypotheses about the evolution of the weights during the tutoring session. We suppose that, when the student answers a specific question asked by the system, the expert module should be able to classify the answer beyond the mere partition right–wrong. More precisely we suppose that the inference engine supplies a m-ple where each element is associated with one of the concepts of procedures involved in the exercise. Each element rates the student's answer in terms of the associated concepts or procedures. Thus, we assume that there are n possible answers associated with each question and, among these, i answers are to be considered as right to some extent ($0 < i < = n$) and j answers as wrong ($j = i - n$). The expert module associates with each right answer an integer k whose maximum value is i, and with each wrong answer an integer k whose minimum value is j. We want that, after each question, the fuzzy weight is increased or decreased according to the correctness or incorrectness of the student's answer.

To allow dynamic evolution to the above-mentioned framework, we have to introduce a suitable function ranging in the interval [0, 1] whose arguments are the fuzzy weight w and the answer value (i or j) and whose behavior is increasing if the answer is right and decreasing if the answer is wrong.

We note the functions root n-th and power n-th satisfy these requirements and moreover their behavior is such that it can truly be viewed as the student's learning curve. Other models in literature present similar functions[9]: Chen and Kurt explicitly suggest modifying the student's learning level "by dilatation" or "by concentration", and this is equivalent to our approach, but they do not care to compute the positive or negative steps carried out by the student.

This discussion about the properties of the function leads us to the following.

Definition. The fuzzy weight w is recursively defined as follows:

$$w':(w,k) \rightarrow \begin{cases} w^{1/2k} & 0 < k < = i \\ w^{2|k|} & j < = k < 0 \quad w \in (0, t) \\ w^{|k|} & j < = k < 0 \quad w \in [t, 1) \end{cases}$$

where w is the current value of the fuzzy weight, n is the overall number of answers, i and j are the maximum and minimum values as regards right and wrong answers, respectively, and k is the value associated with the concept present in the exercise.

As regards the meaning of the parameter t, we note that the interval $[t, 1)$ indicates the range of values for which the student is assumed to have mastery of a specific concept. Moreover, we stress that the function w' in the interval $[t, 1)$ does not exhibit a symmetric behavior: this circumstance is correct since several noise factors, e.g. careless mistakes, bias the results of an exercise and it is worth not emphasizing their influence when the fuzzy weight is beyond a threshold value. As regards the range of values for the parameter t, we observe that in the interval $[0.7, 0.9]$, for each of the lines, one can recognize a value for t from which onwards the derivative is next to zero, and this mathematical feature models adequately the fact that, when a certain level of knowledge is achieved, further slight variations must affect partially the overall assessment.

In a similar way, we have to fix appropriate ranges of values for the weight w, each corresponding to a learning level. To this aim, the interval $[0, 1]$ can be partitioned into three sub-intervals:

Level 1 $w \in (0, s)$ (no knowledge)
Level 2 $w \in [s, t]$ (the system is unable to assess)
Level 3 $w \in (t, 1)$ (knowledge)

Finally we have to discuss what a reasonable range of values for the parameter s is. We note that starting from the value 3.5 the derivative of the power function takes values next to zero whereas the derivative of the root function takes high values: this fact models adequately the fact that if the learning level of a concept is low and a right answer occurs the level should be increased to a reasonable extent, whereas a wrong answer can only slightly affect a situation already compromised.

4. A HEURISTIC FOR STUDENT MODELING

In the following we suppose that the topic to be taught includes concepts and related procedures. This is not a limiting feature and allows, for mathematical topics, one to achieve effective and practical management of the student's knowledge. The student module consists of three submodules:

Knowledge: this submodule contains all the topics learnt by the student
Characteristics: this stores peculiar traits of the student
History: this includes all relevant information about the specific teaching path.

We focus our attention on the Knowledge Module since the others are not affected by our fuzzy approach and can be developed in a traditional way. The task of the Knowledge Module is to store the student's knowledge level about the topic to be learnt. This level has to be managed dynamically by means of the fuzzy measure. The structure of the Knowledge Module is depicted in Fig. 1.

We can consider two distinct classes: conceptual knowledge and procedural knowledge. In each class, PaneA contains the knowledge mastered by the student, PaneB contains concepts and procedures whose mastery has not been achieved by the student, and PaneC includes student's

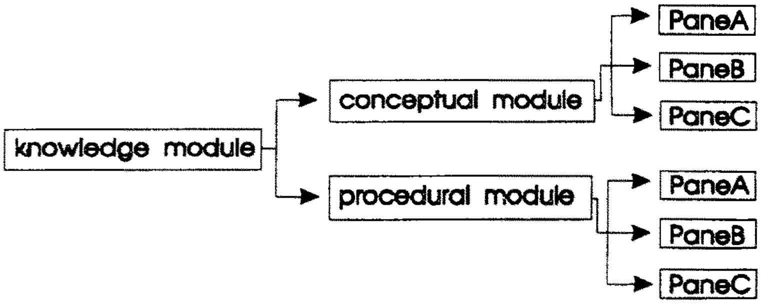

Fig. 1. The internal architecture of the knowledge module.

mal-knowledge. In all panes is also present, associated with each concept or procedure, the fuzzy weight w.

To illustrate the heuristics which drive knowledge acquisition and its cancellation from the panes, let us consider the following example. Suppose that the student is asked to measure the perimeter of a right-angled triangle starting with the measurements of its three sides. If the student's answer is wrong, the student might be not good at figures or does not know the concept of perimeter. In general, the student will get the correct answer or a wrong one according to the pattern depicted in Table 1.

Our heuristic deals in the same way with the two cases in which the right answer is gotten because the probability that the student gets the right answer several times starting from wrong assumptions is very low. The system investigates to some extent whether the right result is achieved by chance and only when the value of the fuzzy weight $w'(w, k)$ exceeds the threshold value is student mastery assumed. Wrong calculations and careless mistakes are treated by the heuristics such that the evaluation is only partially affected by their occurrence. However, the case in which a wrong answer occurs because of an incorrect choice of procedure receives most attention since this is when a misconception occurred.

For instance, consider the above-mentioned example and assume that the lengths of the catheti (the two shorter sides) equal 7 and 4. Then if the student answers incorrectly that the perimeter equals 14 then the system releases control to the expert module in order to recognize the keywords present in the exercise. They are *perimeter*, *triangle*, *right-angled triangle*, *cathetus*, *hypotenuse* and *Pythagoras' theorem*. Thus, the inference engine starts considering rules and mal-rules, facts and non-facts and, starting from the student's answer, tracks backward the chain which has led to the wrong answer. If one path is recognized, panes are suitably updated, otherwise other exercises are suggested by the system. In case the system is unable to back track then the whole exercise is stored in a specific module and will be subsequently examined by the human expert to enlarge the inferential abilities of the system. In our example, the system realizes that the procedure *AreaMeasurement* has been applied in the wrong circumstance.

This heuristic, as applied to the concepts $c_1, c_2, \ldots, c_h$ and to the procedures $p_1, p_2, \ldots, p_z$, can be stated in general terms as follows:

1. Activate the engine for the concepts and procedures $(c_1, c_2, \ldots, c_h, p_1, p_2, \ldots, p_z)$.
2. *If* only one path is gotten
   then compute the fuzzy weight w'.
   Store the mal-rule or the non-fact in PaneC.
   For the remaining c_r (or p_y),
   if the corresponding value $w \in [t, 1)$, then c_r (or p_y) are stored in PaneA;
   if $w \in [s, t)$, then c_r (or p_y) are stored in PaneB

Table 1. Analysis of student responses

Procedure	Answer	Remarks
Right	Right	OK
Right	Wrong	Wrong calculation, careless mistakes
Wrong	Right	Very unlikely
Wrong	Wrong	Conceptual or procedural mistake

3. *If* several paths are recognized
   then Suggest new exercises involving concepts and procedures related to the paths
   else Store the whole exercise into the specific module and do not draw any conclusion

This heuristic is self-explaining except as regards the selection of new exercises and this matter will be discussed in the next section.

5. THE TEACHING STRATEGY

The exercises to be suggested to the student certainly are chosen depending on tutoring goals to be achieved, yet they depend also upon concepts and procedures stored in PaneB. Thus, exercises are stored in a library whose directory contains the keywords related to each exercise. The selection process is driven by a module which operates in forward-chaining mode starting from concepts that are facts and procedures that are the union of several rules. Then the teaching module, by means of the information present in PaneA, PaneB and PaneC, recognizes the exercise to be suggested.

Consider an example. Suppose that we want to decide whether a careless mistake or a conceptual error occurred when the student incorrectly answered that the perimeter equals 14. In this case the forward module can suggest a similar exercise with different sides. The concepts and the procedures involved in the new exercise are a subset of those related to the old one. The unique constraint is that only one of them will not belong to PaneA of the student; thus possible sources of error are, step by step, taken in consideration.

In general, let **Fi** be the known facts, corresponding to concepts, let **FDi** be the inferred facts and let **Ri** be a set of rules, corresponding to a procedure, and place in round brackets the particular instance of a concept or a procedure. A tree is built up by the module, as shown in Fig. 2, where rules are represented by leaf nodes and known and infered facts by other nodes: nodes and branches are labeled by means of the used instances. In the tree, known facts represent the hypotheses of the problem, leaf nodes represent the thesis. Of course, if concepts and procedures present in the PaneC are considered, this happens only for the arguments: the concept or procedure are drawn by the corresponding object knowledge.

In the above-mentioned example, one has:

F1 = given fact—concept of right-angled triangle.
F2(3) = given fact—concept of cathetus whose length equals 3.
F3(5) = given fact—concept of hypotenuse whose length equals 5.
FD1(4) = deduced fact—length of the cathetus = 4.
FD2(12) = deduced fact—measurement of the perimeter = 12.
R1(5,–,3) = union of several rules leading to the Pythagoras' theorem, where the hypotenuse equals 5 and a cathetus equals 3.
R2(3, 4, 5) = rule for measuring a three-sided polygon.

As depicted in Fig. 2, concepts and procedures are drawn from the knowledge base with the only provision that only one concept or procedure can be not present in PaneA.

In such a way, starting from known facts (right-angled triangle whose hypotenuse equals 5 and whose cathetus equals 3) the conclusion is inferred that the perimeter equals 12. This will be the exercise which will be suggested or in explicit form—"Prove that, given a right-angled triangle

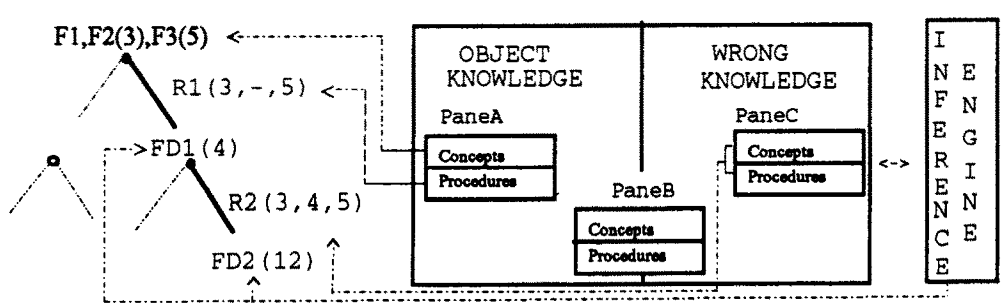

Fig. 2. Selecting the exercises.

whose hypotenuse and cathetus equal 5 and 3, respectively, the perimeter measurement equals 12" or in interrogative form—"Given a right-angled triangle whose hypotenuse and cathetus equal 5 and 3, respectively, what is the measure of the perimeter?".

Of course this is only one among many possible exercises: in fact, the instances are pseudo-random and moreover the forward module uses several rules and consequently many leaf nodes are obtained each corresponding to a different exercise.

Now we have to consider that an exercise can be suggested as the final part of a lesson or to help the student to overcome some difficulties. In both cases the system recognizes a subset of concepts and procedures, and generates the exercises according to the above sketched guidelines. However, in the first case the keywords are stored in advance by the human expert whereas in the second case the keywords are related to the exercise answered incorrectly by the student.

As regards the procedure to select the most appropriate exercise among those generated we note that when the tutoring goal is to improve the student's knowledge concerning a specific concept, exercises containing a minimum of other concepts are to be chosen. However, this minimum number depends upon the mean value of the fuzzy weights w related to n taught topics. Thus we have to take in account the quantity $C_g = \Sigma w_i / n$ which measures the student's overall learning level: the value taken by this function also affects the choice of the exercises since more difficult ones will be suggested in the presence of high values of the function.

6. CONCLUSION

We are aware that the work so far described is only a starting point for the introduction of non-traditional logics to deal with the student's cognitive state. Yet we think that the fuzziness inherently present when the student's learning level is to be evaluated, merits further scrutiny of this approach. The strategy for student modeling is based on a heuristic procedure yet, if the effective management of the student's knowledge is the primary goal, it is practical and feasible. The teaching module can benefit also from this approach.

The first-level prototype of the student module based on the fuzzy weight, which dynamically manages the student's learning level, is undergoing field-testing in various mathematical areas and the preliminary results are rather encouraging.

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